

Linear Algebra:

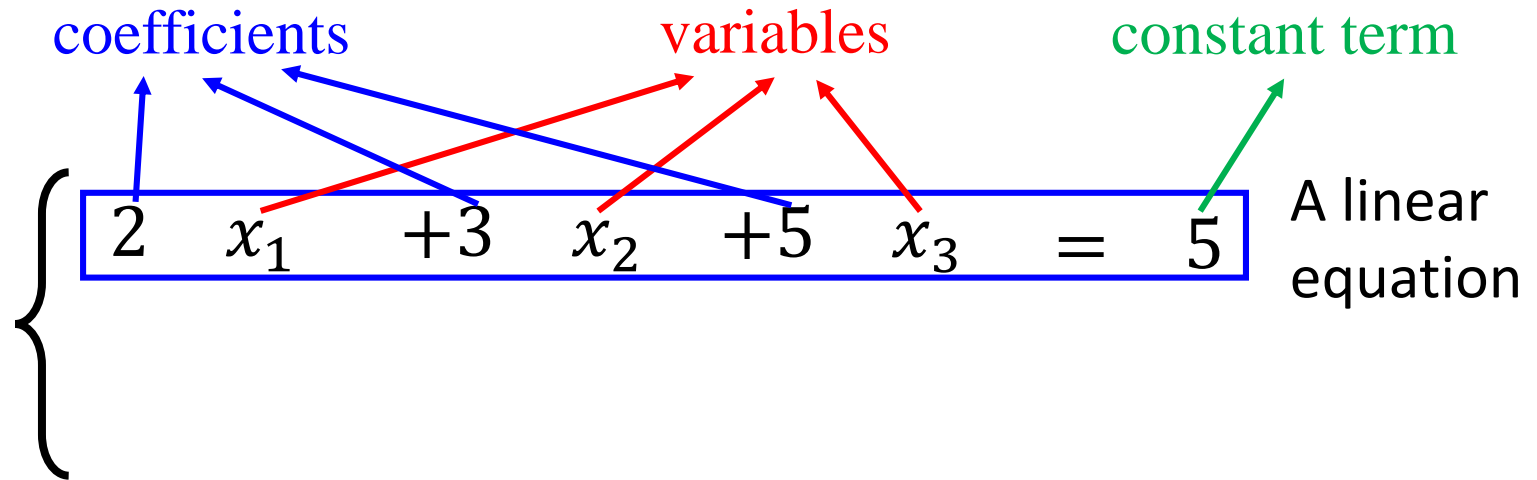
What are we going to learn?

李宏毅

Hung-yi Lee

What are we going to learn?

- A system of linear equations



a system of linear equations

I believe you know how to solve it.

What are we going to learn?

- A system of linear equations (多元一次聯立方程式)

$$\begin{array}{l} \text{m} \\ \text{equations} \end{array} \left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{array} \right.$$

n variables

In this course, m and n can be large

What are we going to learn?

- Solving a system of linear equations

$$\begin{array}{rcl} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & = & b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & = & b_2 \\ \vdots & & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & = & b_m \end{array}$$

Annotations: "span" (yellow), "Independent" (blue), "rank" (green)

Does it have solution?

Approximate Solution

Does it have unique solution?

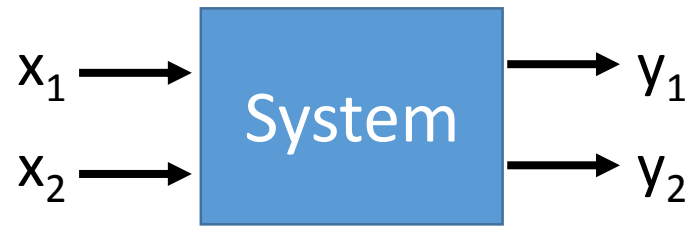
How to find the solution?

Determinants (行列式)

Beyond 3 X 3

Different view from high school

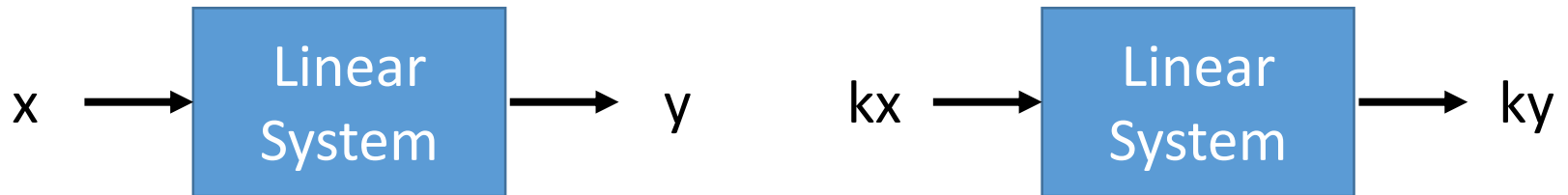
Linear System



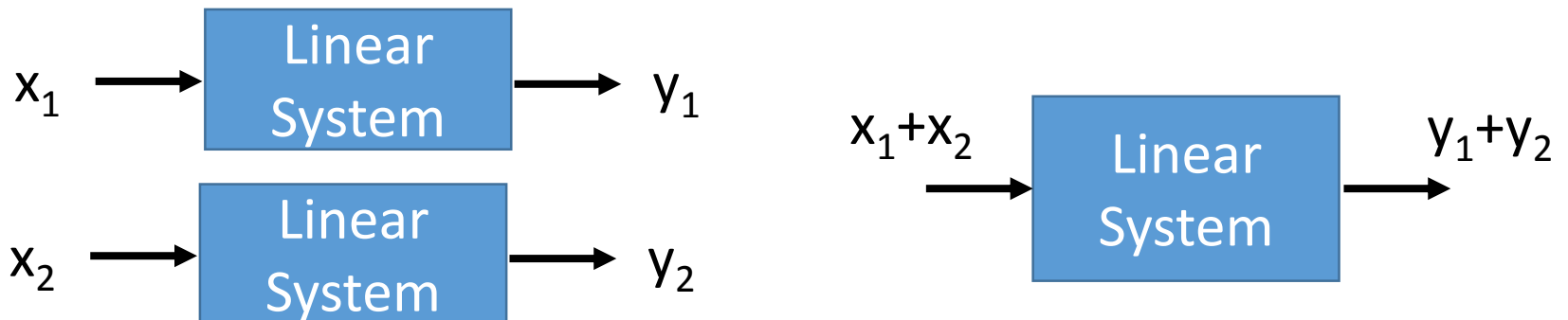
- We use a system of linear equations to describe a linear system
- What is system?
 - function, transformation, operator
 - A system has input and output
 - E.g. Speech Recognition System: The input is audio signals, and the output is recognition results.
 - E.g. Siri: You say “Hi, Siri”, it reply “What can I do for you?”
 - E.g. Communication system
 - Can have multiple inputs and outputs

Linear System

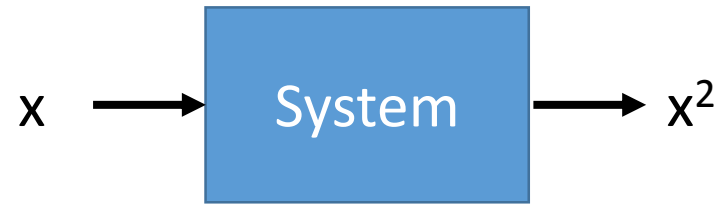
- Linear system have two properties
 - 1. Persevering Multiplication



- 2. Persevering Addition

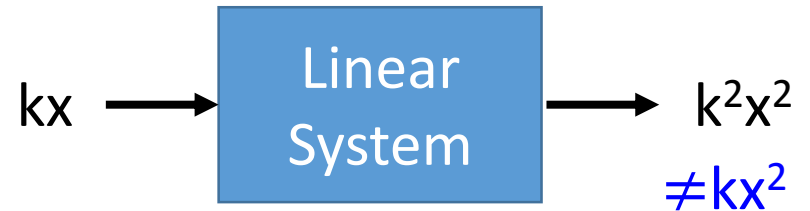
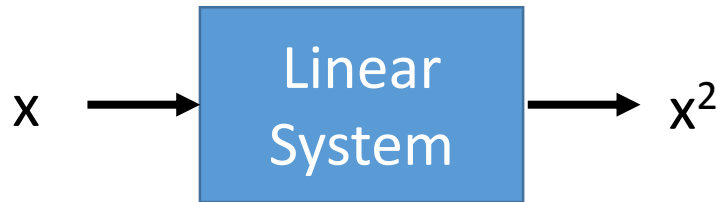


Linear System

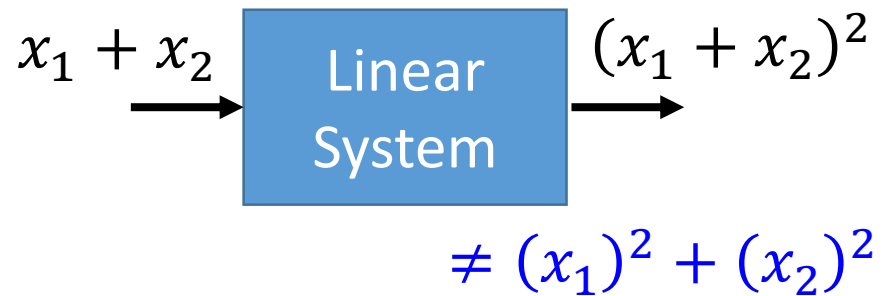
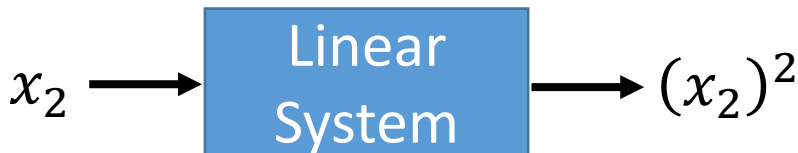
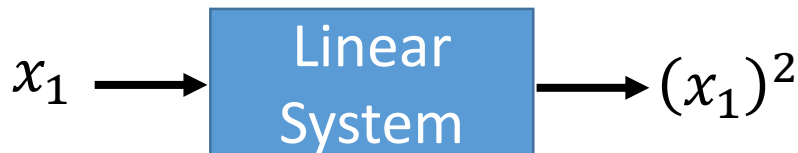


- Linear system have two properties

- 1. Persevering Multiplication



- 2. Persevering Addition

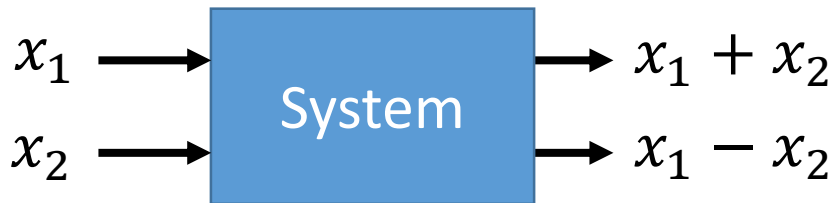


Linear System

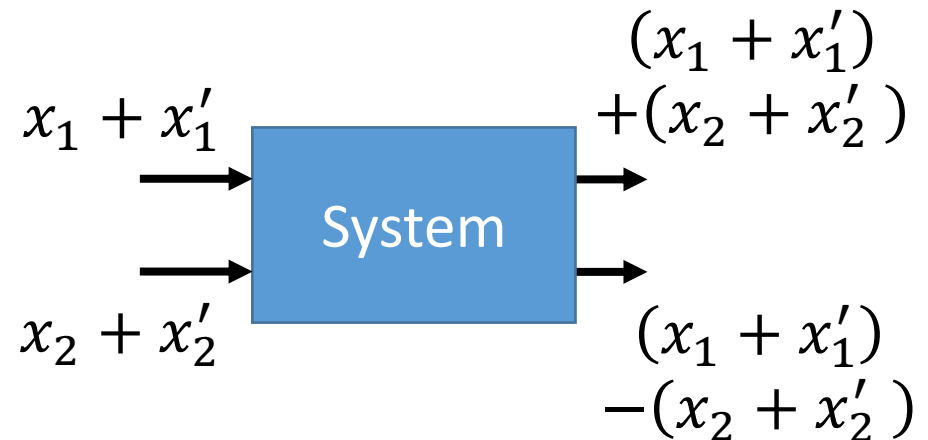
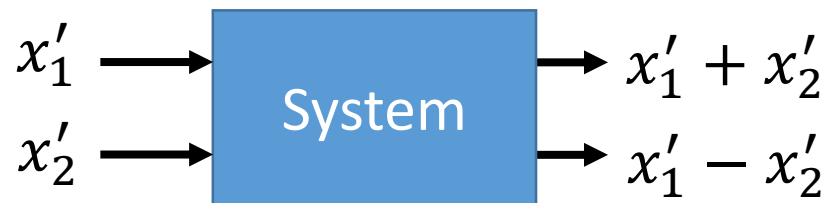
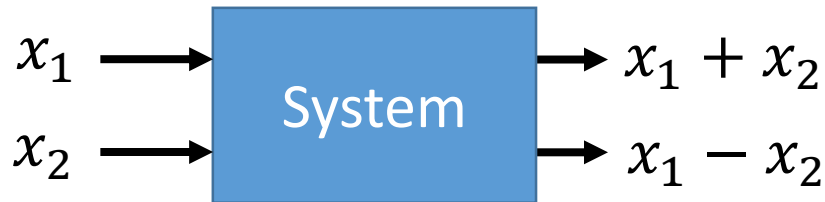


- Two properties

- 1. Persevering Multiplication

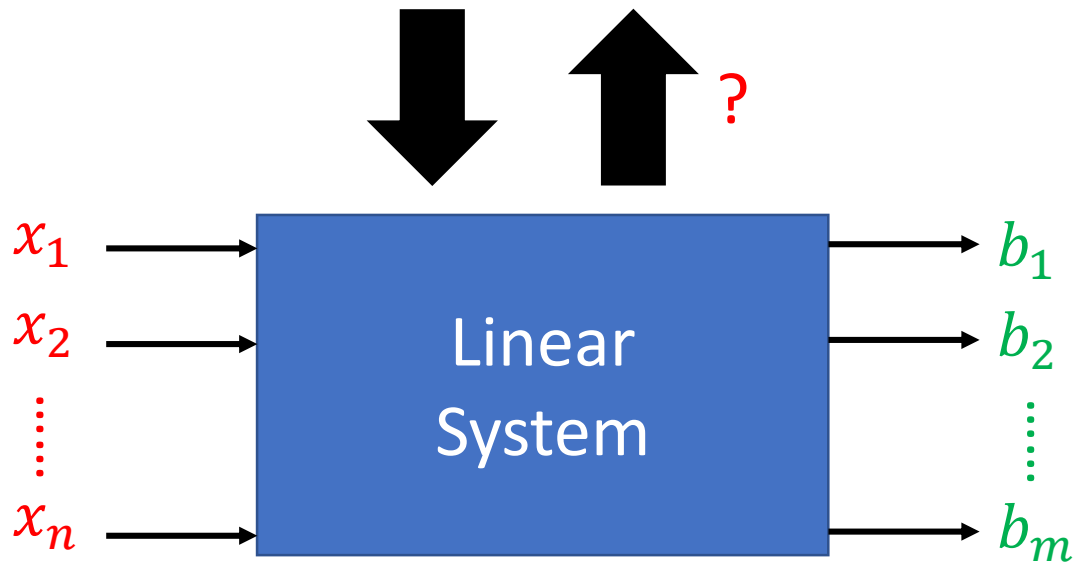


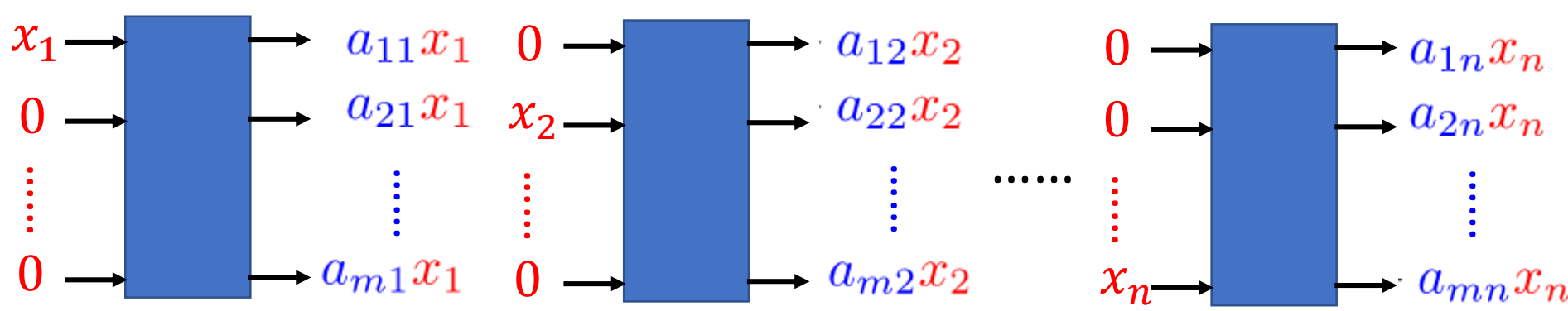
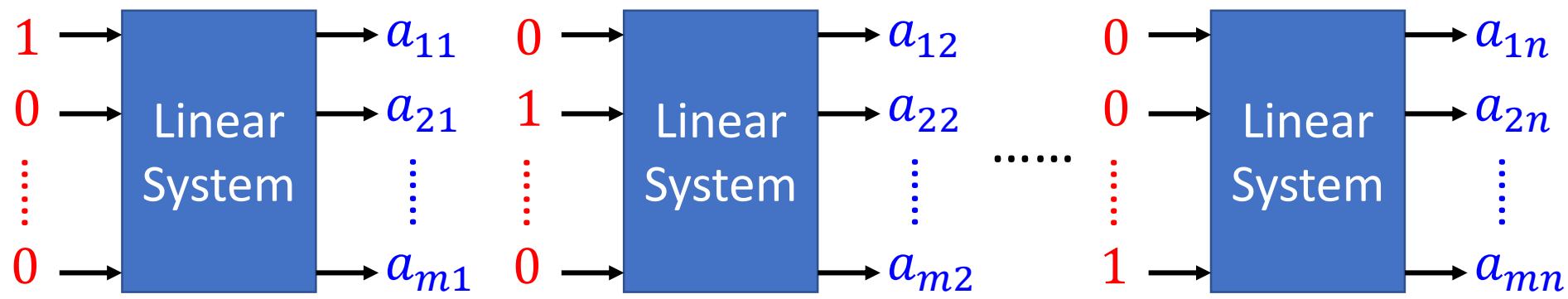
- 2. Persevering Addition



Linear System v.s. System of Linear Equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$





A single linear system is shown with all input variables x_1, x_2, \dots, x_n . The outputs are the full linear equations for each row.

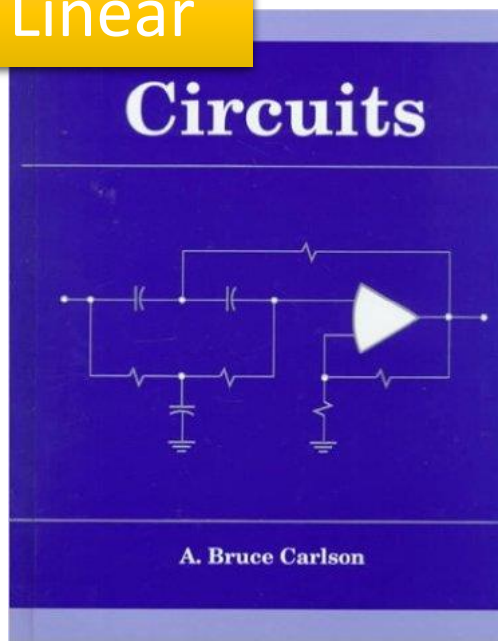
$$\begin{aligned}
 x_1 \rightarrow & a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\
 x_2 \rightarrow & a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\
 & \vdots \\
 x_n \rightarrow & a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m
 \end{aligned}$$

A linear system is described by a system of linear equations

Applications

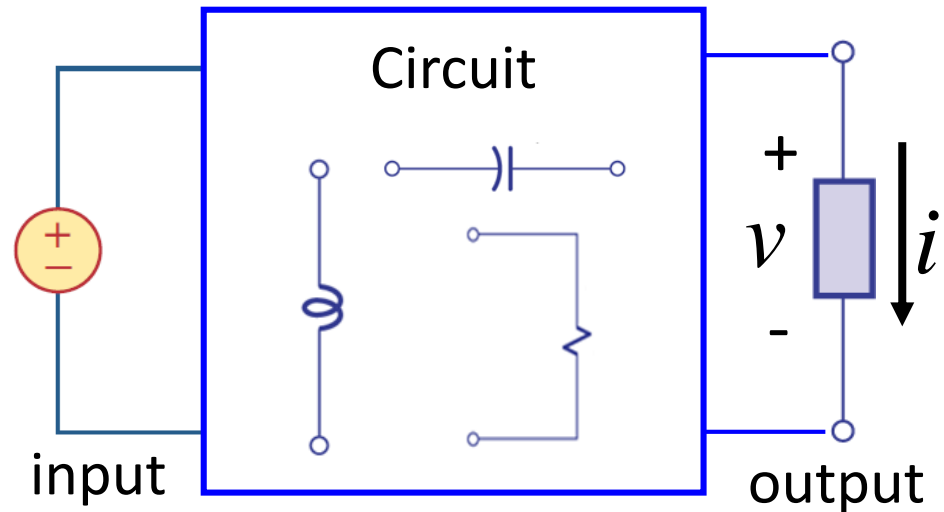
Most system are (assumed to) be linear

Linear



(大二上)

Input: voltage source, current source
output: voltage and current on the load (燈泡、引擎)

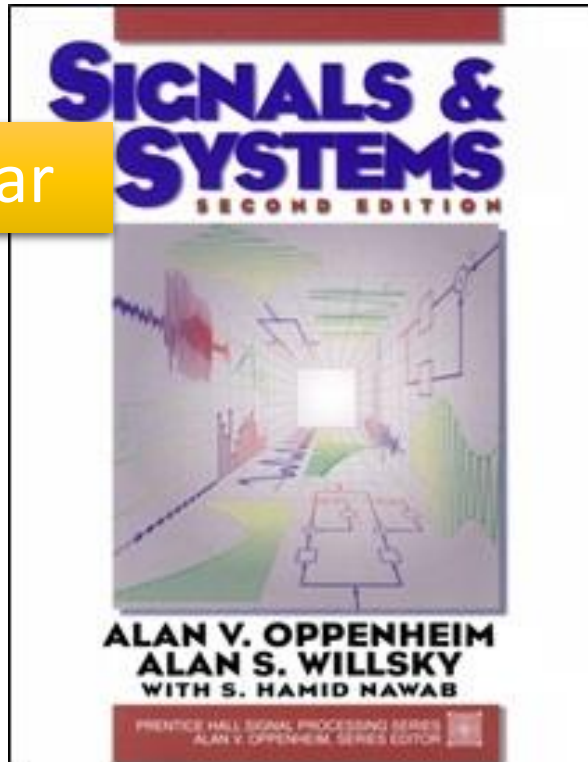


Linear System

Applications

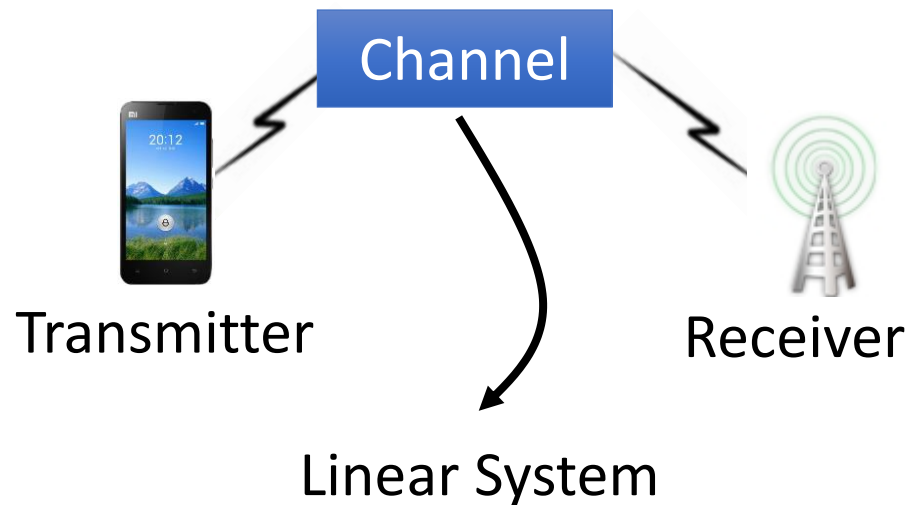
Most system are (assumed to) be linear

Linear



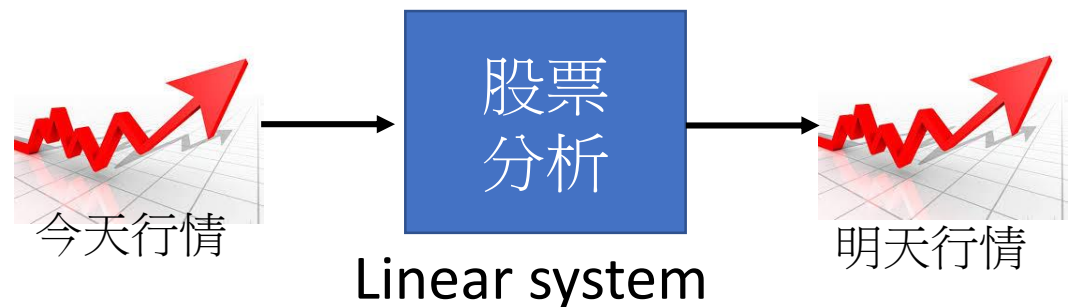
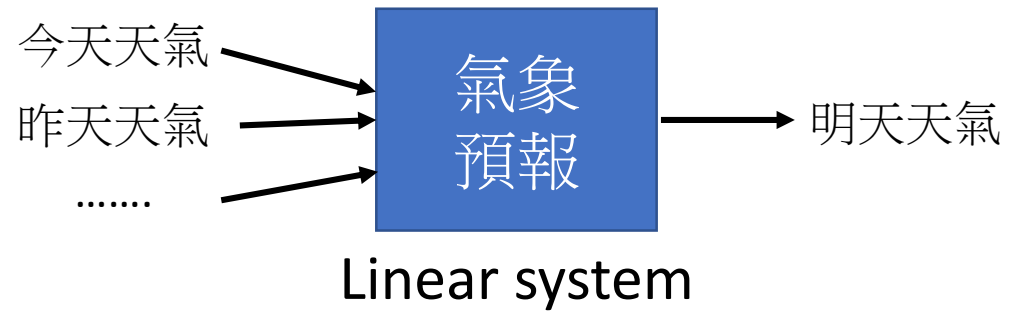
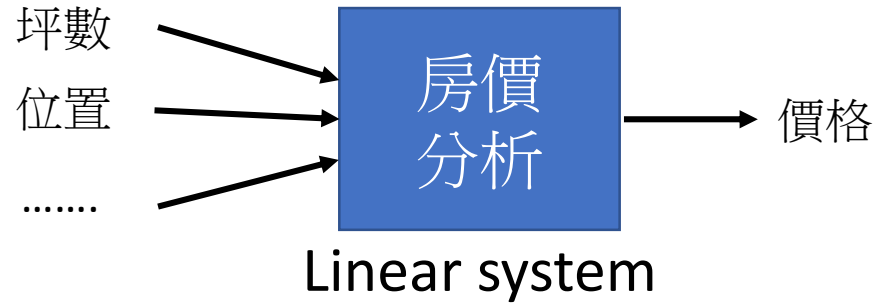
(大二下)

Communication System



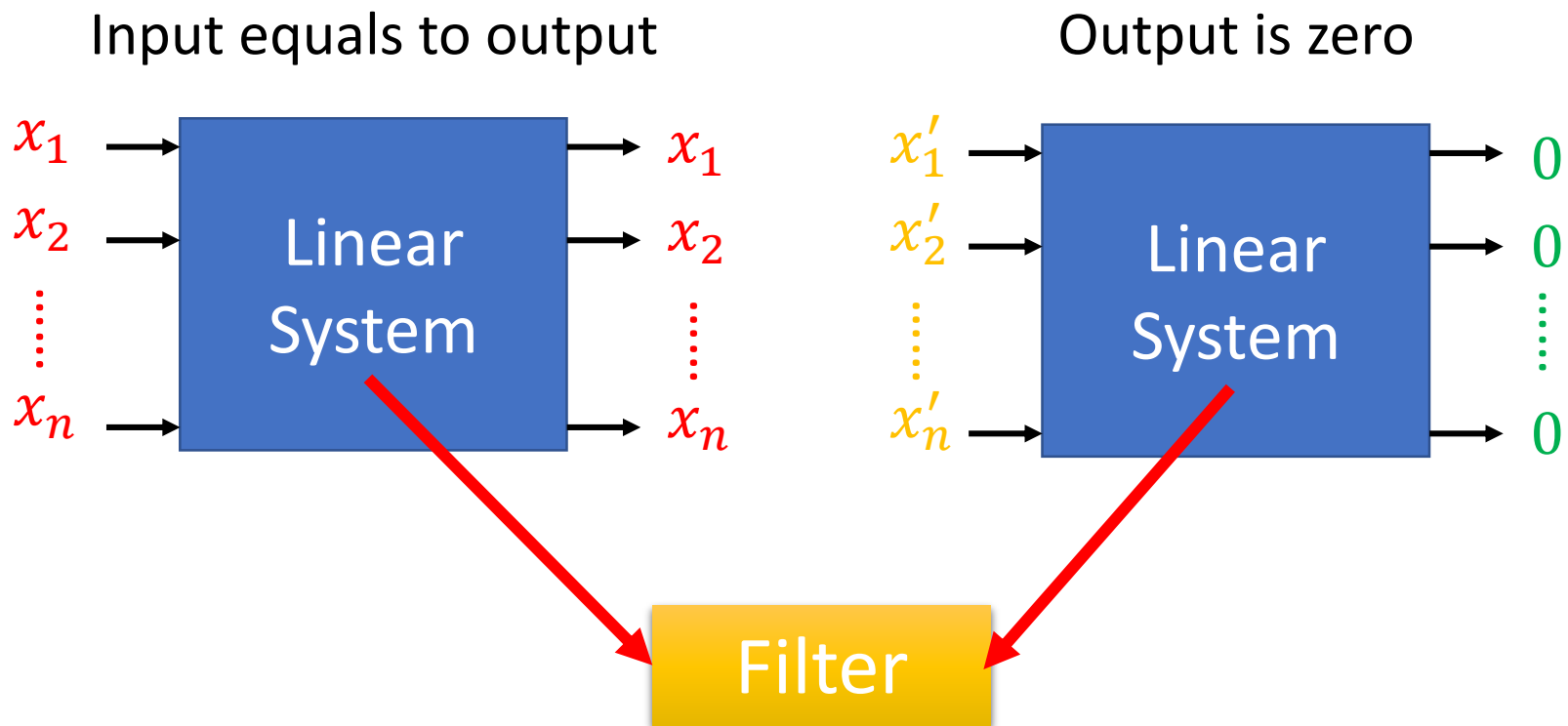
Applications

- Machine Learning
 - E.g. Prediction



Applications

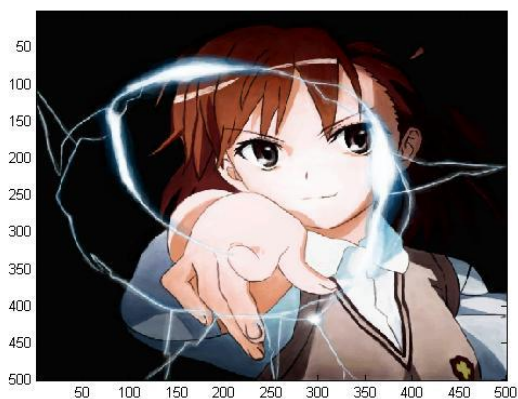
- We will learn EigenXXXXX



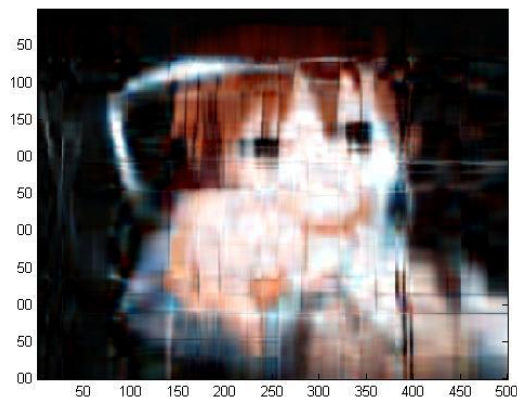
Applications – Image Compression

壓縮比: 25

壓縮比: 12.5

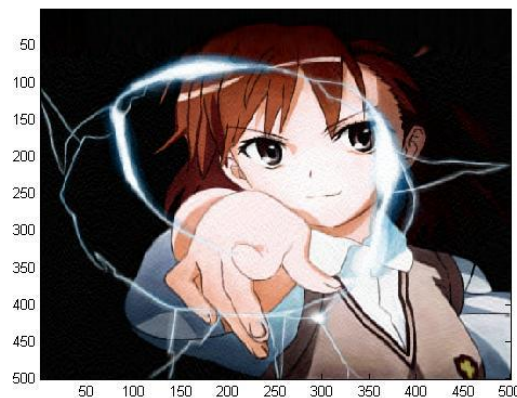
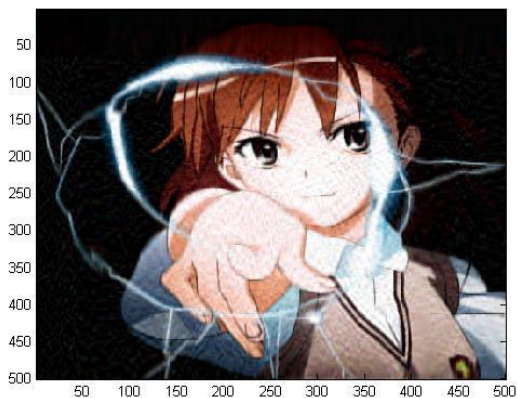


原圖

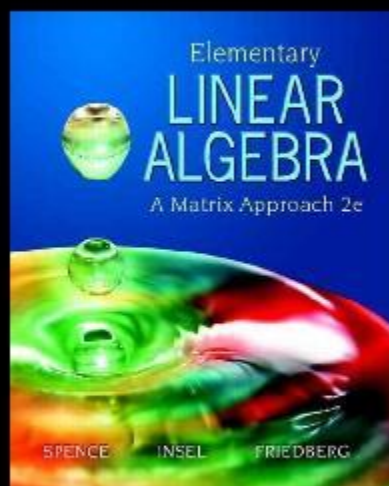


壓縮比: 5

壓縮比: 2.5



Linear Algebra is Important



如果沒把線代學好

卻上了大二

就好像沒有學念能力

卻到了天空鬥技場兩百樓

李宏毅